

String Field Theories from One Matrix Models

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Abstract

Through the continuum limit of the one matrix model on the multicritical point the corresponding Schwinger-Dyson equation of temporal-gauge string field theory is derived. It agrees with that of the background independent formulation recently proposed.

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Discrete methods, realised analytically by matrix models, have been instrumented in furthering our understanding of string theory. For example, it is known that the double scaling limit[1] of the one matrix model on the k -th critical point[2] corresponds to a $(2, 2k-1)$ type string[3]. Recently important progress has been made in this direction via the formulation of temporal-gauge string field theory in the $c = 0$ [4] and $c = 1 - 6/m(m+1)$ [5, 6] cases. The corresponding W constraints[7] expected from matrix models can be derived from the appropriate continuum Schwinger-Dyson equations. This approach is motivated by the transfer matrix formalism outlined in [8] which was later applied to the multicritical case[9]. These theories and multicritical points of the one matrix model were studied via a background independent formulation in [10]. These considerations were extended to closed and open strings in [11] and [12].

That such models are indeed connected with string theory was verified when dynamical triangulation was shown to yield $c = 0$ string field theory in the continuum limit[13]. Stochastic quantization of the matrix model was also considered in [14]. In addition to these studies the continuum Schwinger-Dyson equations for $c = 0$ and $c = 1/2$ were derived from the one and two matrix ϕ^3 model, respectively[15]. In this letter we wish to use the techniques of [15] to derive the continuum Schwinger-Dyson equation of the k -th critical point of the one matrix model and confirm the results first presented in [10].

Let us consider the one matrix model. We take the matrix ϕ to be an $N \times N$ hermitian matrix. In order to obtain the k -th critical point we define the action as

$$S(\phi) = N \text{tr} \left(\sum_{n=2}^{k+1} \frac{\lambda_n}{n} \phi^n \right). \quad (1)$$

By rescaling ϕ we choose to set $\lambda_2 = 1$. The loop operator of length n is defined as

$$W(n) = \begin{cases} \frac{1}{N} \text{tr} \phi^n & (n \geq 0) \\ 0 & (n \leq -1). \end{cases} \quad (2)$$

With this operator we define the partition function of loop amplitudes in the matrix model to be

$$Z_m(J) = \frac{\int d\phi e^{-S-S_J}}{\int d\phi e^{-S}}, \quad (3)$$

where

$$S_J(\phi) = - \sum_n J(n) W(n). \quad (4)$$

In this approach the Schwinger-Dyson equation is expressed by setting the integration of the total derivative to be zero:

$$\int d\phi \frac{1}{N^2} \text{tr} \left(\frac{\partial}{\partial \phi} \phi^{n-1} \right) e^{-S-S_J} = 0. \quad (5)$$

Here $\partial/\partial\phi$ operates not only on ϕ^{n-1} but also on S and S_J . When $n \geq 1$ this equation can be expressed in terms of $Z_m(J)$. Using the function $\theta(n)$ defined as

$$\theta(n) = \begin{cases} 1 & (n \geq 1) \\ 0 & (n \leq 0), \end{cases} \quad (6)$$

we can rewrite the above equation as

$$\left[\sum_{m=-\infty}^{\infty} \frac{\partial^2}{\partial J(m) \partial J(n-2-m)} + \frac{\theta(n)}{N^2} \sum_{m=1}^{\infty} m J(m) \frac{\partial}{\partial J(n-2+m)} \right. \\ \left. - \sum_{m=2}^{k+1} \lambda_m \frac{\partial}{\partial J(n-2+m)} + \sum_{i=0}^{k-1} \delta_{n+i,0} \sum_{m=i+2}^{k+1} \lambda_m \frac{\partial}{\partial J(-i-2+m)} \right] Z_m(J) = 0. \quad (7)$$

In the continuum limit the δ -function and its derivatives, expressed in terms of the loop length, will appear in $W(n)$. In order to obtain the continuum loop operator we have to subtract these terms from $W(n)$. We therefore employ the following partition function Z_c :

$$Z_m(J) = \exp \left(\sum_n J(n) c(n) \right) Z_c(J), \quad (8)$$

where $c(n)$ consists of Kronecker δ 's. This $c(n)$ can be obtained by demanding that the linear terms of $\partial/\partial J$ vanish in eq.(7) expressed now via Z_c . Therefore

$$c(n) = -\frac{1}{2} \sum_{m=2}^{k+1} \lambda_m \delta_{n+m,0}, \quad (9)$$

from which eq.(7) becomes

$$\left[\sum_{m=-\infty}^{\infty} \frac{\partial^2}{\partial J(m) \partial J(n-2-m)} + \frac{\theta(n)}{N^2} \sum_{m=1}^{\infty} m J(m) \frac{\partial}{\partial J(n-2+m)} \right. \\ \left. - \sum_{m=-\infty}^{\infty} c(m) c(n-2-m) + \sum_{i=0}^{k-1} \delta_{n+i,0} \sum_{m=i+2}^{k+1} \lambda_m \frac{\partial}{\partial J(-i-2+m)} \right] Z_c(J) \\ = 0. \quad (10)$$

Note that since $W(0) = 1$ we can replace $\partial/\partial J(0)$ in the last term of eq.(10) by 1. For simplicity we choose to drop $\partial/\partial J(m)$ when $m \geq 1$ in the last term by multiplying eq.(10) by $n(n+1) \cdots (n+k-2)$. We further define

$$J_c(n) = y_c^{-n} J(n), \quad (11)$$

so that the loop operator becomes $y_c^n (W(n) - c(n))$. Therefore eq.(10) can be rewritten as

$$n(n+1) \cdots (n+k-2) \left[\sum_{m=-\infty}^{\infty} \frac{\partial^2}{\partial J_c(m) \partial J_c(n-2-m)} + \frac{\theta(n)}{N^2} \sum_{m=1}^{\infty} m J_c(m) \frac{\partial}{\partial J_c(n-2+m)} \right. \\ \left. - y_c^{n-2} \sum_{m=-\infty}^{\infty} c(m) c(n-2-m) + y_c^{n-2} \lambda_{k+1} \delta_{n+k-1,0} \right] Z_c(J_c) = 0. \quad (12)$$

In the continuum limit we let the length of each side of the loop tend to zero, $a \rightarrow 0$, with the loop length held fixed, $l = na$. In this limit we have

$$\sum_n \rightarrow \frac{1}{a} \int dl, \quad (13)$$

$$\theta(n) \rightarrow \Theta(l), \quad (14)$$

$$\delta_{n+i,0} \rightarrow a\delta(l+ia). \quad (15)$$

Let us find the k -th critical point of this model. Now that there are k parameters, $y_c, \lambda_3, \lambda_4, \dots, \lambda_{k+1}$, we can choose the critical values of these so that when we expand the last two terms of eq.(12) in a each coefficient of a, a^2, \dots, a^k will vanish. Since the last two terms of eq.(12) are of order a^{k+1} on this critical point and eq.(12) holds for every order of a , the first two terms should also be of order a^{k+1} . Therefore from the first term we find

$$\frac{\partial}{\partial J_c(n)} \sim a^{k+\frac{1}{2}} \frac{\delta}{\delta j(l)}, \quad (16)$$

where $\delta/\delta j(l)$ creates the continuum loop operator:

$$Z_c(J_c) \rightarrow \left\langle \exp \left(\int_0^\infty dl j(l) w(l) \right) \right\rangle \equiv Z[j]. \quad (17)$$

Eq.(16) means that

$$J_c(n) \sim a^{-k+\frac{1}{2}} j(l), \quad (18)$$

and in order to make the second term of eq.(12) of the same order as the other terms we set

$$\frac{1}{N^2} = a^{2k+1} g, \quad (19)$$

where g is the string coupling constant[1]. The order a^{k+1} contribution of eq.(12) then yields the continuum Schwinger-Dyson equation:

$$\left[l^{k-1} \left\{ \int_0^l dl' \frac{\delta^2}{\delta j(l') \delta j(l-l')} + g \int_0^\infty dl' l' j(l') \frac{\delta}{\delta j(l+l')} \right\} + C \delta^{(k)}(l) \right] Z[j] = 0, \quad (20)$$

where $l \geq 0$ and C is some constant. Note that there is a factor of l^{k-1} which admits the k -th critical point[10]. This equation is the one on the critical point. If the parameters approach the critical point as

$$\lambda_m = \lambda_m^{critical} \left(1 + \sum_{n=2}^k A_{m,n} a^n \right), \quad (21)$$

and if we choose $A_{m,n}$ so that the coefficient of a^3, a^4, \dots, a^k in the last two terms of eq.(12) vanish, then the terms proportional to $\delta^{(k-2)}(l), \delta^{(k-3)}(l), \dots$ will be added to eq.(20). By further choosing $A_{m,n}$ appropriately this equation will become the case of non-zero cosmological constant. If we drop the factor of l^{k-1} in eq.(20) more terms proportional to the δ -function or its derivative will appear. The terms of this kind have already been derived in [10]. They can be determined by requiring that the continuum Schwinger-Dyson equation becomes the Virasoro constraints with shifted variables.

In this letter we have shown that the k -th critical point of the one matrix model becomes string field theory whose Schwinger-Dyson equation have the factor of l^{k-1} . This agrees with [10]. If we drop this factor we will have to deal with terms like $\delta_{n,0} \partial/\partial J(m)$ which were not considered here. This kind of analysis may be necessary for higher critical points of the two matrix model which correspond to the $c = 1 - 6/m(m+1)$ string.

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